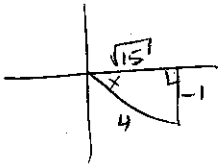
Simplify the following expressions completely. Show proper reasoning to justify your answer. **SCORE: ____ / 12 PTS**

[a] $\cot(\arcsin(-\frac{1}{4})) = -\sqrt{15}$

$x = \arcsin(-\frac{1}{4})$

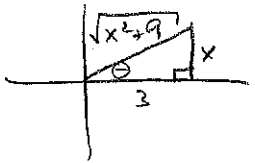
$\sin x = -\frac{1}{4}$ AND $x \in Q_4$



[b] $\csc(\tan^{-1}\frac{x}{3})$, where $x > 0$ $= \frac{\sqrt{x^2+9}}{x}$

$\theta = \tan^{-1}\frac{x}{3}$

$\tan\theta = \frac{x}{3}$ AND $\theta \in Q_1$



Graph 2 periods of the function $y = -6\cos(\frac{2}{3}x + \frac{7\pi}{3}) - 2$.

SCORE: ____ / 16 PTS

Find the coordinates of the 9 points discussed in lecture, corresponding to 2 complete periods, starting at the phase shift.
Label all $x-$ and $y-$ values for the 9 points on the appropriate axes, using a consistent scale for each axis.

MIDDLE $y = -2$

AMPLITUDE $|-6| = 6$

TOP $= -2 + 6 = 4$

BOTTOM $= -2 - 6 = -8$

PERIOD $= \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$

START $\frac{2}{3}x + \frac{7\pi}{3} = 0$

$\frac{2}{3}x = -\frac{7\pi}{3}$

$x = -\frac{7\pi}{3} \cdot \frac{3}{2} = -\frac{7\pi}{2}$

$\frac{1}{4}$ PERIOD $= \frac{3\pi}{4}$

POINTS:

($\frac{-14\pi}{4}$, -8)

($\frac{-11\pi}{4}$, -2)

($\frac{-8\pi}{4}$, 4)

($\frac{-5\pi}{4}$, -2)

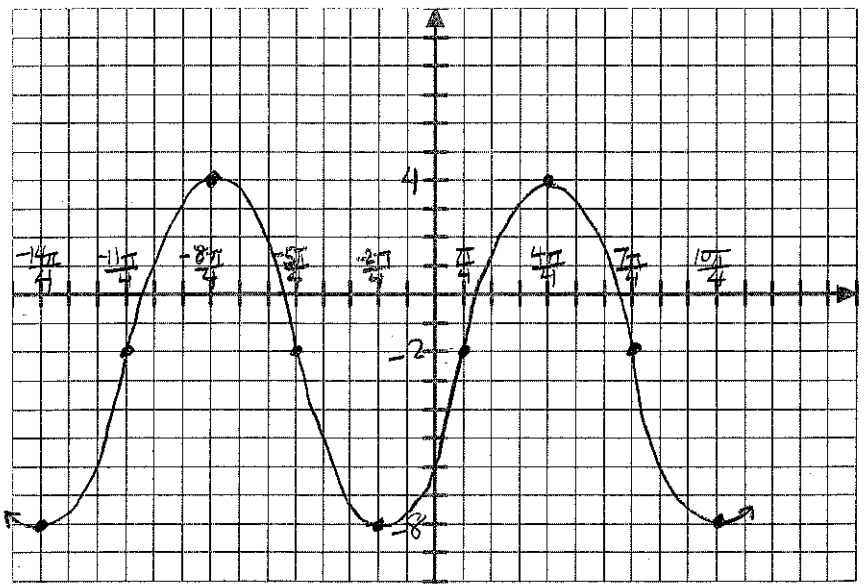
($\frac{-2\pi}{4}$, -8)

($\frac{\pi}{4}$, -2)

($\frac{4\pi}{4}$, 4)

($\frac{7\pi}{4}$, -2)

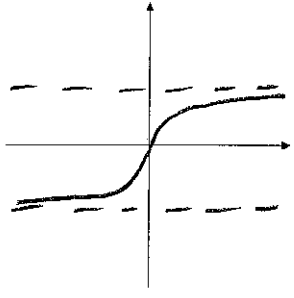
($\frac{10\pi}{4}$, -8)



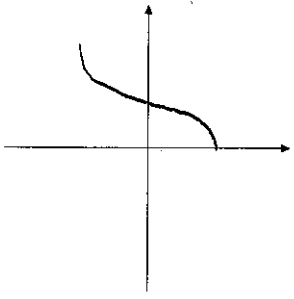
Sketch the graphs. **For periodic functions, sketch at least 2 periods.**
You only need to get the general position and shape correct. Do NOT plot points.

SCORE: ____ / 12 PTS

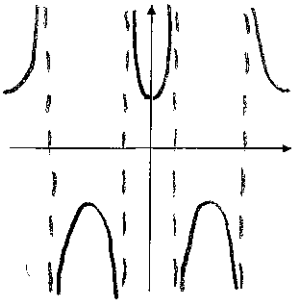
[a] $y = \arctan x$



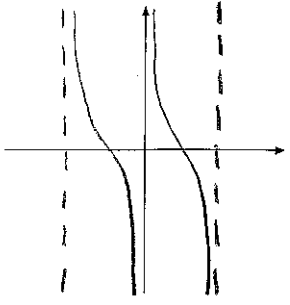
[b] $y = \cos^{-1} x$



[c] $y = \sec x$



[d] $y = \cot x$



Fill in the blanks. Write "DNE" if the question has no answer.

SCORE: ____ / 32 PTS

[a] The range of $f(x)=\sec x$ is $(-\infty,-1]\cup[1,\infty)$.

[b] The domain of $f(x)=\sin^{-1}x$ is $[-1,1]$.

[c] The equations of the asymptotes of $f(x)=\csc x$ are $x=n\pi,\ n\in\mathbb{Z}$.

[d] The equations of the asymptotes of $f(x)=\arctan x$ are $y=\pm\frac{\pi}{2}$.

[e] $\cos^{-1}(-\frac{\sqrt{2}}{2})=\frac{3\pi}{4}$.

[f] $\arctan(-1)=-\frac{\pi}{4}$.

[g] $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$.

[h] $\arcsin\frac{1}{2}=\frac{\pi}{6}$.

[i] $\tan(\arctan 3)=3$.

[j] $\arccos(\cos\frac{5\pi}{4})=\frac{3\pi}{4}$.

[k] $\cos^{-1}(\cos\frac{5\pi}{6})=\frac{5\pi}{6}$.

[l] $\sin(\sin^{-1}3)=$ DNE.

[m] The domain of $f(x)=\tan x$ is $x\neq\frac{\pi}{2}+n\pi,\ n\in\mathbb{Z}$

[n] The range of $f(x)=\arccos x$ is $[0,\pi]$.

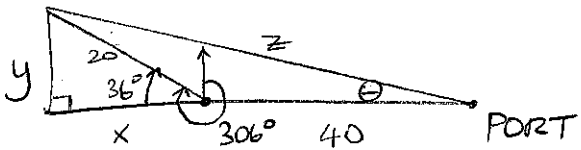
[o] As $x\to 0^-$, $\cot x\to -\infty$.

[p] As $x\to\frac{\pi}{2}^+$, $\sec x\to -\infty$.

A ship leaves port at noon and heads due west at 20 nautical miles per hour. **SCORE: ____ / 14 PTS**

At 2pm, the ship changes course to a bearing of 306°.

[a] How far is the ship from port at 3pm ? (Round your answer to the nearest integer.)



$\cos 36^\circ = \frac{x}{20} \rightarrow x = 20\cos 36^\circ \approx 16.18$

$\sin 36^\circ = \frac{y}{20} \rightarrow y = 20 \sin 36^\circ \approx 11.76$

$z = \sqrt{(40+16.18)^2 + 11.76^2} \approx 57.4$

57 NAUTICAL MILES

[b] What is the bearing of the ship from port at 3pm ? Your final answer should be a single number (ie. "bearing of _____ °"). (Round your answer to the nearest degree.)

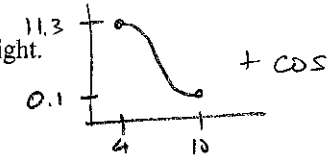
$\tan\theta = \frac{11.76}{40+16.18} \approx 0.2093$

$\theta \approx \tan^{-1}(0.2093) \approx 12^\circ$ BEARING $\approx 270^\circ + 12^\circ = 282^\circ$

The depth of the water at the end of a dock is a sinusoidal function. **SCORE: ____ / 14 PTS**

At 4am, the water has its maximum depth of 11.3 feet, and the depth decreases until, and at 10am, the water has its minimum depth of 0.1 feet.

[a] Find an equation for the depth of the water at t hours after midnight.



MIDDLE $= \frac{11.3+0.1}{2} = 5.7 = D$

AMPLITUDE $= \frac{11.3-0.1}{2} = 5.6 = |A|$

PERIOD $= 2(10-4) = 12 = \frac{2\pi}{B} \rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$

START $= 4$

$5.6 \cos \frac{\pi}{6}(t-4) + 5.7$

[b] Find the depth of the water at 2pm. (Round your answer to 1 decimal point.)

$t = 12+2 = 14$

$5.6 \cos \frac{\pi}{6}(14-4) + 5.7$

$= 5.6 \cos \frac{5\pi}{3} + 5.7$

$= 5.6\left(\frac{1}{2}\right) + 5.7 = 8.5$ FT